Dynamic System Synthesis in Terms of Bond Graph Prototypes

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This paper deals with two synthesis methods for dynamic systems as an application of the bond graph prototypes which are originally developed for dynamic system analysis in the frequency domain. The first method called the analytical synthesis uses the procedures similar to the network synthesis in the electrical field and an already-existing one, but yet demonstrates its own strengths inherited from the bond graph prototypes such as the freedom from the causality assignment and determination of junction types. The second method called the direct synthesis is introduced in this paper to provide physical realization for a given specification of impedance and admittance forms without any mathematical manipulations except a simple division. The two synthesis methods are shown through examples in each case to be a concise method in their usage and a scientific method in managing the bond graph prototypes systematically.

Key Words: Bond Graph Prototypes, Feedforward and Feedback Expansions, Analytical Synthesis Method, Direct Synthesis Method

1. Introduction

Dynamic system design and/or synthesis have been typically concerned with the parameter selection and optimization for existing configurations of idealized system components. It may be said that the design of a dynamic system involves with questions such as what the system is to do and how it can be done, while the synthesis is chiefly associated with the implementation of a system function to provide its desired response and eventually the physical structure by which the system function can be exactly realized in any single or multiple energy domains.

This paper attempts to make the bond graph prototypes (Park and Kim, 1997) stronger and more effective in synthesizing a dynamic system. The bond graph prototypes have been proposed by the authors to reduce bond graph structures without any alteration of physical similarity and to find some useful properties in the frequency domain (i. e., relative degrees, zero and characteristic dynamics, transfer functions etc.) directly from the reduced bond graph. Therefore, the methods presented in this paper can naturally be thought of as the reverse ones used in dynamic system analysis in the sense that the latter focuses on the reduction of a given bond graph model, while the former the expansion (or the reticulation) of a given system function into smaller components whose physical equivalents are easily recognized (see Fig. 1).

As the definitions of impedance and admittance (Park and Kim, 1997), the given specifications are assumed to be linear functions with constant coefficients and those consisting only of passive elements for applying the bond graph prototypes equally to both analysis and synthesis of dynamic systems in a dual manner. However, nonlinear components may, if required, replace their nonlinear relationships with linear constitutive laws after the synthesis is completed. And the synthesis on systems containing active elements has currently been studied since the invention of operational amplifiers.

Not much work is found on the investigation of dynamic system synthesis in terms of bond

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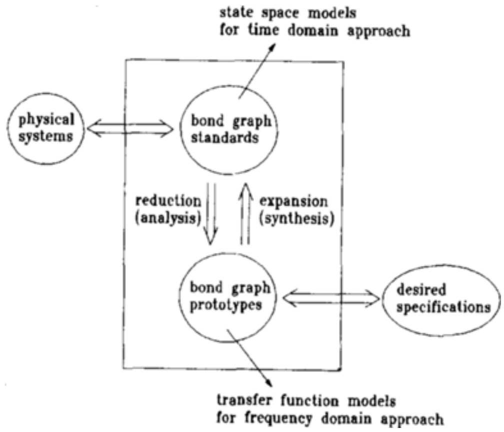

Fig. 1 Dynamic system analysis and synthesis diagrams with respect to bond graphs.

graphs. Notable exceptions within the limit of our knowledge are Redfield and Krishnan (1993). In their researches, more than one bond graph structure to meet a single dynamic specification were presented particularly for the conceptual design of a system which has originally developed to generate design concepts or objectives with relatively ease even in the face of configurational constraints. However, their procedure to synthesize a dynamic system is principally based on some views about the signal flow in block diagrams rather than the power flow in bond graphs. Thus, the method still possesses problems in selecting the causality and determining what types of the bond graph junctions are best physically reasonable.

In this paper, two synthesis methods are developed by using the bond graph prototypes, the analytical synthesis and the direct synthesis. We show that the analytical synthesis provides some troubleshootings for problems left in the approach suggested by Redfield et al. And through the direct synthesis, in particular, the completed synthesis for dynamic systems can be obtained with no mathematical manipulations except simple division, and the method gives some clear information for linking between the analysis and synthesis of dynamic systems. Furthermore, we will try to explain physically the significant notions found in each step of synthesizing dynamic systems as possible as we can.

The remainder of this paper is organized as follows: Section 2 discusses electrical network synthesis and its bond graphs; section 3 some useful properties on system functions given a frequency domain input-output specification of impedance or admittance forms. And the analytical and direct synthesis methods are developed in section 4 by treating the bond graph prototypes as a tool for expanding a given specification systematically. Conclusions are offered in section 5.

2. Network Synthesis and Bond Graphs

System synthesis is the opposite of analysis. Rather than break a system into parts, it usually combines parts into a system. But at the position of bond graphs, we may say that bond graph analysis is the reduction of the Paynter's bond graphs (called in this paper the bond graph standards) into the bond graph prototypes and bond graph synthesis is the expansion felt as the reverse manner as shown in Fig. 1. Thus, it can be natural to say that a procedure for bond graph synthesis, if developed, should be accomplished by the opposite of bond graph analysis. In other words, the use of the bond graph prototypes as a medium allows the procedure to be more credible in the logical sense. However, we first review briefly in this section the two famous works on network synthesis commonly used in the electrical field in order to assist the development of our methods with some useful expansion techniques.

Electrical network synthesis is the generation of electrical circuits and networks to meet a desired specification in the frequency domain. Of some practical methods, we focus mainly on those employing partial or continued fraction expansion techniques by which a specified impedance or admittance can be broken into smaller components until each function of the components is recognized as representing physical configurations of a circuit element. As discussed earlier, the network may of course either consist of purely passive elements or contain active devices. We but consider in here only the problem of determining a passive network to give a specified system function. In other words, the function's behavior is assumed to be achieved by passive systems whose transfer functions are termed "positive real." Since passive systems, for example, systems consisting of only inductors, resistors, and capacitors in the electrical system and of only masses, dampers, and springs in the mechanical system, require no active or external source of power, they must be characterized by energy dissipation related to the input-output behavior and stability. A good introduction to the concept of positive real is in Anderson (1973) and Tomlinson (1991), and we will present some useful properties on positive real system functions in the following section to begin dealing with dynamic system synthesis using bond graph prototypes.

One method to approach the problem of synthesis of a network to realize a specified system function, which is a positive real and rational function, is based on a partial fraction expansion (PFE) of the function. For example, a system function F(s) with natural frequencies from zero to infinity consecutively can be reticulated by the PFE into the form

$$F(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2 + \omega_i^2} + k_\infty s \tag{1}$$

In Eq. (1), the first and last terms represent poles at zero and infinity, respectively, which can be realized directly by a capacitor C and an inductor L, simply by inspection of impedances and admittances for passive elements of bond graphs. And the second terms represent n pairs of imaginary poles which can also be realized by the string of the parallel LC combinations. Thus, if F (s) is the form of impedance, then Eq. (1) implies that each term is combined together with the 1-junction to merge the outputs of the capacitor, inductor, and the string of LC combinations. Figure 2(a) and (b) show the realization of Eq. (1) and its bond graph, respectively. In specific, the structure like that of Fig. 2(a) is called as Foster realizations.

Another method to approach for synthesizing system functions is based on a continued fraction expansion (CFE) by extracting a pole of the function at infinity and inverting the remaining function sequentially. The CFE method can be clearly explained with a ladder network, or bond graphs containing structures such as *pi's* and tees (Karnopp et al., 1990). Consider Fig. 3(a) and (b) that represent a ladder network and its bond graph with a tee structure, respectively. Referring to the tee network, the specified impedance $Z(s)$ at the driving-port is given by Z_1 in series with the impedance to the right of Z_1 and the latter is equal to the inverse of the admittance to the right of Z_1 which is given by Y_2 in parallel with the admittance to the right of Y_2. Similarly, the admittance to the right of Y_2 is equal to the inverse of the impedance to the right of Y_2 and this impedance is given by Z_3 in series with the impedance to the right of Z_3. Continuing in this way the $Z(s)$ can be expanded as

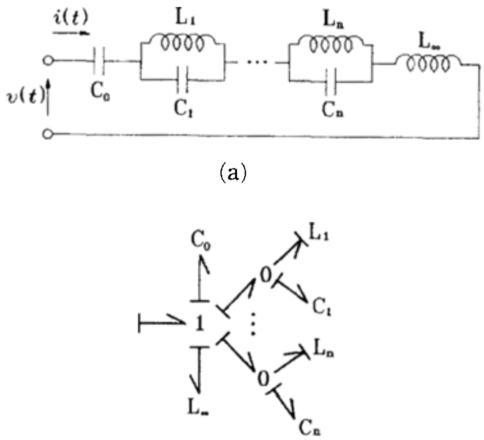

Fig. 2 Foster realization (a) and its bond graph (b).

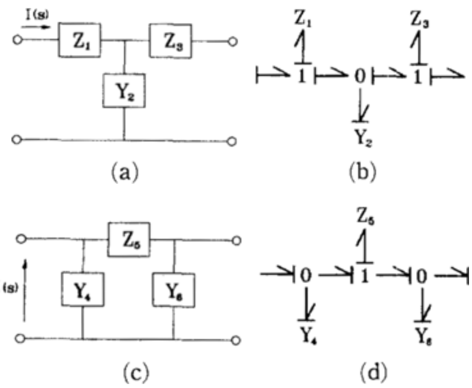

Fig. 3 Bond graphs with *tee* and *pi* structures.

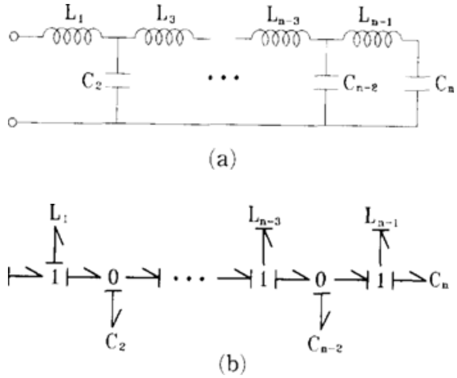

Fig. 4 Cauer realization (a) and its bond graph (b).

$$Z(s)=Z_1+\cfrac{1}{Y_2+\cfrac{1}{Z_3+\cdots}} \tag{2}$$

For the *pi* network which starts with a shunt element as shown in Fig. 3(c) and (d) the specified admittance $Y(s)$ can be expressed as

$$Y(s)=Y_4+\cfrac{1}{Z_5+\cfrac{1}{Y_6+\cdots}} \tag{3}$$

Once again, if we specify the impedance specification with natural frequencies of whole ranges as Eq. (2), then the first pole at infinity is introduced by an inductor with the derivative causality and all the remaining poles can also be made by dynamic elements with the integral causality such as a chain of capacitor and inductor pairs because of the nature of inversion. Figure 4 (a) shows the realization of Eq. (2) which is well known as Cauer realizations, and its bond graph is involved in Fig. 4(b). It is also said that for Eq. (3) the same realization and bond graph as in Fig. 4 are build up except for starting with the parallel element instead of with the series element.

The work of this paper is to extend the above two schemes of network synthesis to the bond graph domain. With the bond graph prototypes, it can be shown that the reduction of a given bond graph and the expansion of a given specification are straight linked each other. Therefore, the bond graph prototypes can be appraised to become a versatile means in analyzing and synthesizing dynamic systems.

3. Some Useful Properties of Passive Systems

As discussed earlier, the conditions that must be satisfied by a rational function which is realized as the driving-point impedance or admittance of a passive system are described in the form known as the positive real conditions. Therefore, we examine in this section some of the inherent properties of a positive real function to make use of them effectively in synthesizing dynamic systems in terms of bond graph prototypes. These properties are reviewed by Anderson (1973) and Tomlinson (1991) in the context of network theory and Slotine (1991) in the analysis of nonlinear systems, and so on.

System functions $F(s)$ of a passive system must mathematically be positive real which is expressed by

$$\mathrm{Re}[F(s)]\geq 0 \text{ for all } \mathrm{Re}[s]\geq 0 \tag{4}$$

Geometrically, Eq. (4) means that the rational function $F(s)$ maps every point in the closed right half including the imaginary axis of the complex plane into the closed right half of the $F(s)$ plane, i. e., the real part of the magnitude of $F(s)$ is greater than or equal to zero when the real part of s is greater than or equal to zero, and the phase shift of $F(s)$ in response to a sinusoidal input is always remained between -90° and 90°. These features are straightforwardly incorporated into the Nyquist plot of $F(j\omega)$. As a rule, the conditions that the Nyquist plot of a function exists in the closed right-half plane impose on the function utterly important or practical constraints such that all the roots of the numerator and denominator polynomials of the rational function $F(s)$ must be in the left-half plane. This is also known as passivity conditions of a system function which generates no energy but dissipates it. According to the Routh-Hurwitz stability criterion, all the coefficients of both the numerator and denominator polynomials must be positive real values in order that their roots (the zeros and poles of the system function) are in the left-half plane. This implies simple necessary conditions for asserting whether a given system function $F(s)$ is realized only by passive elements: $F(s)$ is stable and minimum-phase. In addition, by recalling the procedure for constructing Nyquist frequency response plots, the relative degree which is defined as the difference between the order of the denominator and that of the numerator of the $F(s)$ must be between plus and minus unity so that the Nyquist plot remains in the right-half plane at high frequencies.

To make the above properties on a positive real function useful for synthesizing dynamic systems using bond graphs, we now inspect the relations between these properties and a realizable function representing the behavior of components in the bond graph structures. In fact, the realizable function should be produced in each step of reticulation from a system function which is specified in the form of impedance or admittance, so it still remains positive real to ensure the conditions like that the function is stable and minimum-phase, and its relative degree is between -1 and 1. At this point, it must be emphasized that the first two requirements can be verified independently by the investigation of whether the numerator and denominator polynomials of a function are the so-called Hurwitz polynomial which satisfies the Routh-Hurwitz stability criterion. But the last indicates the mutual dependency of numerator and denominator polynomials. This means that the prescribed knowledge of one of the two polynomials determines the form of the other. Accordingly, synthesizing a positive real function into components which is also positive real is perfectly controlled by whether the information easily recognizable enough to apply other polynomial is possible or not. Fortunately, we can see the relative degree of the driving-point impedance or admittance by inspection when the bond graph prototypes are used in synthesizing dynamic systems. In case of the bond graph prototypes with feedforward paths alone, the relative degree of a system function is taken as the smallest one in the feedforward bonds. And the relative degree for the bond graph prototypes with both feedforward and feedback paths is equal to that of components in the feedforward bonds (Park and Kim, 1997). These two observations on the bond graph prototypes will be basically used in this paper, in particular, to develop the analytical synthesis method. However, the main contribution of this paper is the direct synthesis where the procedure of dynamic system synthesis is performed directly on the bond graph prototypes with no help of the above observations and in addition any mathematical handling in choosing the coefficients of numerator and denominator polynomials.

4. Dynamic System Synthesis Using Bond Graph Prototypes

The bond graph prototypes can apply to the analysis of dynamic systems in the frequency domain effectively, just as having shown their applications through a passive tuned isolator as an example in the paper (Park and Kim, 1997). Where the transfer function of a system in question can be obtained directly from its physical structure of the bond graph. Now, we develop in this section the two procedures for synthesizing bond graphs from an already specified function given a impedance or admittance form.

The first method is basically related to circuit synthesis method. Based on the PFE and the CFE and the two observations on relative degrees mentioned in the earlier section, and the zero and characteristic dynamics obtained from the bond graph prototypes, given specifications are broken into component parts such that these components remain positive real to ensure a passive system. In particular, if the specification involves any feedback structure this method is more adequate in a theoretical or analytical viewpoint than that introduced by Redfield and Krishnan (1993).

The second method is the cardinal point of this paper, which will make the bond graph prototypes more applicable to dynamic system synthesis as well as modeling and analysis of dynamic systems. This method is performed from the direct manipulation of the bond graph prototypes, without the selection of appropriate parameters the first method must necessitate. Then we will call the first method as the analytical synthesis and the second as the direct synthesis for effective representation.

4.1 The analytical synthesis

It is generally known that the behavior of all dynamic systems can roughly be represented by the combinations of feedforward and feedback structures in their corresponding physical domains. We also know that the bond graph techniques are widely used not only in the hardware level at which the physical realization of a system can be implemented more comfortably than the other pictorial ones such as block diagram method and signal flow graph, but also in the abstract level at which the manipulation for finding the mathematical descriptions of a system such as transfer function models and state space models can be practicable without loss of physical similarity. This is the main reason why the bond graph is used in the analysis and synthesis of dynamic systems. Synthesizing a given specification into bond graphs is simply reduced to the reticulation of the desired function into components until each of their components can be suitably represented by basic 1-port elements and then completely assembled together with the best selected junctions of bond graphs, at which time some fundamental laws of physics such as continuity and compatibility equations must be not violated. However, it sometimes needs the help of an excellent engineer armed with rich knowledge about physical intuitions for dynamic systems and implicit topologies covered in the bond graph techniques. These difficulties in synthesizing dynamic systems are now resolved simply in this and following sections by introducing the bond graph prototypes

Figure 5 shows the bond graph prototypes where they are classified with the feedforward and feedback categories with respect to a given specification of admittance and impedance forms. If given specifications are expanded only into the feedforward terms, then they can be formed by summing the terms in Fig. 5(a) and (c) as follows:

$$SY_{ff}(s)=\sum_{i=1}^{n}Y_i(s)=Y(s)+1/Z(s) \qquad (5)$$

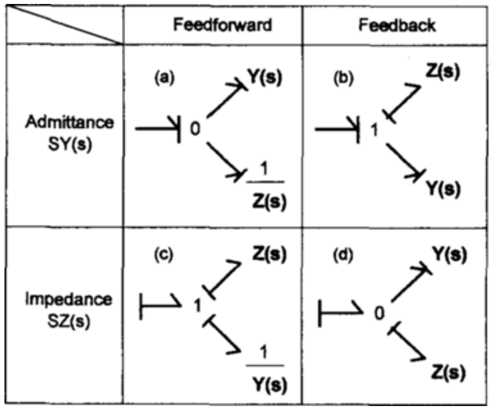

Fig. 5 The bond graph prototypes for feedforward and feedback expansions.

$$SZ_{ff}(s)=\sum_{i=1}^{n}Z_i(s)=Z(s)+1/Y(s) \qquad (6)$$

where $SY_{ff}(s)$ and $SZ_{ff}(s)$ denote the driving-point admittance and impedance, respectively, when they both possess the feedforward terms alone. The middle side of Eqs. (5) and (6) imply the general cases having the n^{th} feedforward paths. With the bond graph prototypes, however, only the two terms (or paths) are required to synthesize a given specification as shown in the right side of the equations because the bond graph prototypes treat each term appeared after reticulation as another specification which in turn, unless its physical equivalent is clearly recognized, is split again into two components only. And the inverse forms of Eqs. (5) and (6) have no other meanings than needed in the following section and then simply mean $Y_1=Y(s)$, $Y_2=1/Z(s)$ in Eq. (5) and $Z_1=Z(s)$, $Z_2=1/Y(s)$ in Eq. (6).

Eqs. (5) and (6) suggest that given specifications regardless of the forms of impedance and admittance can be reticulated into partial fraction components or it can be expanded into the sum of parts with a common denominator. Of course, the components must remain positive real for a passive synthesis. Note that with the bond graph prototypes there are no any nuisance in thinking over the selection among junction types because they are already fixed as in Fig. 5. In the impedance synthesis, for example, the 1-junction serves as a feedforward junction since a flow is input from the external bond. And the 0-junction is appropriate for the admittance synthesis.

On the other hand, a given specification can also be broken into both feedforward and feedback parts as shown in Fig. 5(b) and (d). Unlike the reticulation of specifications into feedforward parts only, the feedback expansion is rather complex since the input signal at the driving-port does not take to piece easily in the compact form such as Eqs. (5) and (6). According to the paper (Park and Kim, 1997), the admittance specification $SY_{fb}(s)$ and the impedance specification $SZ_{fb}(s)$ are, respectively, expanded as

$$SY_{fb}(s)=\frac{Y(s)}{1+Y(s)Z(s)} \tag{7}$$

$$SZ_{fb}(s)=\frac{Z(s)}{1+Z(s)Y(s)} \tag{8}$$

where $Z(s)$ in Eq. (7) and Y(s) in Eq. (8) are feedback parts, while $Y(s)$ in Eq. (7) and $Z(s)$ in Eq. (8) are feedforward parts.

The junction types are also automatically assigned even in the feedback expansion as in the feedforward expansion. However, while the approaches to seek the *terms* involved in Eqs. (5) and (6) may be relatively uncomplicated which often use the partial fraction expansion or the so-called common denominator expansion just above mentioned, the *factors* contained in Eqs. (7) and (8) cannot be easily extracted from a given specification $SY_{fb}(s)$ or $SZ_{fb}(s)$ since the equations have a non-unique inversion. For this reason, we recollect some results on system functions obtained from the bond graph prototypes and some observations on relative degrees stated in the earlier section in order that the procedure for the feedback expansion becomes to be more analytical and conceptual one.

If the given impedance $SZ_{fb}(s)$ is reticulated into the two components $Y(s)$ and $Z(s)$ of Fig. 5(d) to provide its physical behaviour in the hardware level, then we can first rewrite the feedback component $Y(s)$ in Eq. (8) in terms of the zero dynamics $SZ_n(s)$ and the characteristic dynamics $SZ_d(s)$ of $SZ_{fb}(s)$ as

$$\begin{aligned}Y(s)&=SZ_{fb}(s)^{-1}-Z(s)^{-1}\\&=\frac{SZ_d(s)Z_n(s)-SZ_n(s)Z_d(s)}{SZ_n(s)Z_n(s)}\end{aligned} \tag{9}$$

where $Z_n(s)$ and $Z_d(s)$ represent the numerator and denominator polynomials of the feedforward component $Z(s)$ respectively. From Eq. (9), it can be known that the determination of $Y(s)$ depends exclusively on how $Z(s)$ may be selected to guarantee the positive real conditions that both $Z(s)$ and $Y(s)$ are stable and minimum phase systems, and in addition the relative orders of them must be between -1 and 1. Note that the relative order of $Z(s)$ is equal to that of $SZ_{fb}(s)$.

One way to solve for $Y(s)$ and $Z(s)$ from a given impedance $SZ_{fb}(s)$ is to eliminate one of the two polynomials $Z_n(s)$ and $Z_d(s)$ in the right side of Eq. (9) so that the order of $Y(s)$ can be reduced to that smaller than the characteristic dynamics $SZ_d(s)$ by deliberately choosing the coefficients of the contributing component after elimination. Based on the fact that the zero dynamics of a given impedance is found in the bond graph prototypes as the feedforward component $Z(s)$, we choose the numerator polynomial $Z_n(s)$ as a factor of $SZ_n(s)$ such that $SZ_n(s)=Z_n(s)SZ_{nr}(s)$ where $SZ_{nr}(s)$ is the remaining factor of $SZ_n(s)$. With this scheme, the $Z_n(s)$ can be cancelled and Eq. (9) becomes

$$Y(s)=\frac{Y_n(s)}{Y_d(s)}=\frac{SZ_d(s)-SZ_{nr}(s)Z_d(s)}{SZ_n(s)} \tag{10}$$

Note that the form of the polynomial $Z_d(s)$ in the right side of Eq. (10) which is not yet known can be inferred from the above scheme and the fact that $Z(s)$ $(=Z_n(s)/Z_d(s))$ has the same relative degree as $SZ(s)$ which is already known. Thus, by choosing the coefficients of $Z_d(s)$ such that the order of $Y_n(s)$ is smaller than that of $SZ_d(s)$, the given impedance $SZ(s)$ can be reticulated into the feedforward component $Z(s)$ and feedback component $Y(s)$, the former having a reduced order compared to $SZ(s)$ and the latter consisting of a numerator whose order is less than $SZ_d(s)$ and the denominator of $SZ_n(s)$.

Now, an example is offered to help understand the analytical synthesis so far explained. Consider a desired function of impedance as

$$SZ(s)=\frac{56s^2+100s+14}{12s^2+15s+3} \tag{11}$$

The form of Eq. (11) is generally used to design passive filters such as notch or band pass types. And the passiveness of this impedance can be easily verified by its stability (poles located at -0.25 and -1) and the energy-dissipative nature the Nyquist plot of Eq. (11) implies which does not trespass the left half of the complex plane at all. To reticulate Eq. (11) into terms or factors which must be recognized by the basic elements of bond graphs for realization in the hardware domain, the PFE is first applied just as the network synthesis in the electrical field has done. But because the orders of the numerator and denominator in Eq. (11) are identical, Eq. (11) is previously divided with synthetic division into

$$SZ(s)=\frac{14}{3}+\frac{10s}{4s^2+5s+1} \quad (12)$$

Applying the PFE to the last term of the right side of Eq. (12) gives

$$SZ(s)=\frac{14}{3}+\frac{10/3}{s+1}-\frac{5/6}{s+1/4} \quad (13)$$

In Eq. (13), the negative term is not positive real and cannot be synthesized passively. Then the first and last terms are combined together for passive synthesis into

$$SZ(s)=\frac{14/3s+1/3}{s+1/4}+\frac{10/3}{s+1} \quad (14)$$

Equations (12) and (14) are the final results of the feedforward expansion, both having only positive real components, and their synthesis in terms of bond graphs are in Fig. 6(a) and (b) respectively. Next, the second term in Eq. (12) and both terms in Eq. (14) are further synthesized by the feedback expansion technique as explained above so that the concluding components can directly be realized by bond graph elements. For the second term of Eq. (12), factoring the numerator $SZ(s)=10s$ into the numerator of the feedforward component $Z_n(s)=10$ and the remaining numerator $SZ_{nr}(s)=s$ yields the denominator of the feedforward components of the form $Z_d(s)=C_1s+C_0$ because the relative degree of $Z(s)$ must be 1, and then makes Eq. (10) to be

$$Y(s)=\frac{(4-C_1)s^2+(5-C_0)s+1}{10s} \quad (15)$$

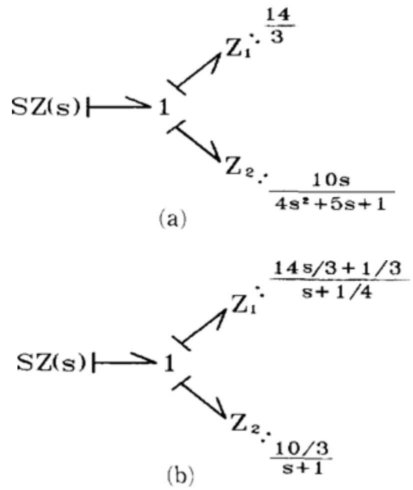

Fig. 6 Bond graph syntheses of Eqs. (12) and (14).

Equation (15) means that the feedback component $Y(s)$ is absolutely determined by how the coefficients of $Z_d(s)$ may be selected. Synthesizing a function can be said to reduce the order of the function continuously. Thus, C_1 is chosen as 4 to cancel the second order term in the numerator polynomial of $Y(s)$, and C_0 is selected to simplify the denominator polynomial of $Z(s)$ as zero. The $Z(s)$ and $Y(s)$ for the feedback expansion of the second term of Eq. (12) can then be searched as follows:

$$Z(s)=\frac{10}{4s} \quad (16)$$

$$Y(s)=\frac{5s+1}{10s} \quad (17)$$

Since the feedback component $Y(s)$ in Eq. (17) shows the same order of the numerator and denominator polynomial, however, it can be again reticulated into two terms simply with synthetic division as

$$Y(s)=\frac{1}{2}+\frac{1}{10s} \quad (18)$$

With these results the analytical synthesis is complete. Eqs. (12), (16) and (18) represent the completed syntheses of the given impedance specification of Eq. (11). Note that with the bond graph prototypes as in Fig. 5 we need not to worry about what types of junctions of bond graph are right at each step of reticulation. Its

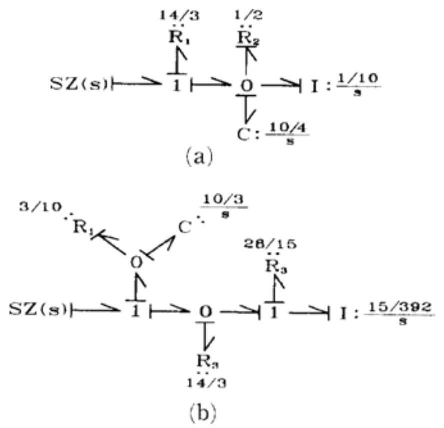

Fig. 7 Complected syntheses of Eqs. (12) and (14).

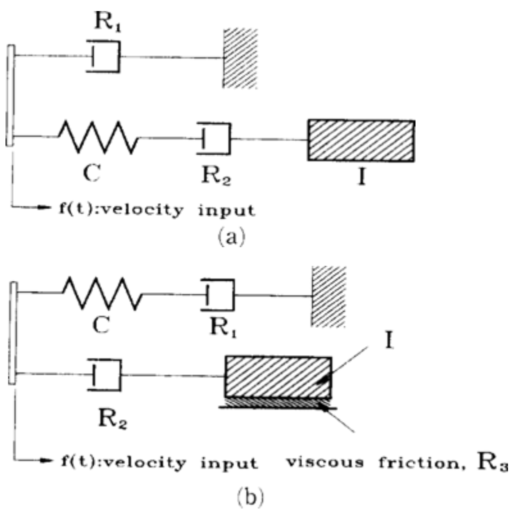

Fig. 8 Mechanical realizations of synthesize bond graph.

work has done very straightforwardly.

Figure 7(a) shows the completed synthesis of Eq. (11) where each component is expressed by the basic elements of bond graph which involve the energy-storage elements of C- and I-types and the energy-dissipative elements of R-type. Another synthesis of Eq. (11) using Eq. (14) rather than Eq. (12) is also appeared in Fig. 7 (b). The procedure generating Fig. 7(b) is exactly the same as that used to deal with the second term of Eq. (12). At this point, it must be emphasized that, whatever different they may be, both structures in Fig. 7 provide the same behavior. In other words, given specifications can be realized as having multiple configurations depending on the path of reticulation as in the above example. The bond graph synthesis, unlike electrical network synthesis, can offer additional reticulation possibilities which may be useful in handling some constraints between a desired specification and the renewable system which is already configured. The physical realizations of Fig. 7(a) and (b) in mechanical domain are possible as in Fig. 8(a) and (b), respectively.

4.2 The direct synthesis

In this section, it is shown that the bond graph prototypes are simple and effective tools for synthesizing a given specification into feedforward and feedback parts. In the previous section, it was presented that the method to find feedforward components only is relatively succinct, while for the feedback expansion the feedforward and feedback components cannot be easily fixed and sometimes involve some delicate problems in obtaining their solutions. According to the reticulating path and parameter selection in feedback and feedforward components, in particular, multiple syntheses are possible such that an automated program is necessarily required to choose one of the whole syntheses in view of optimizing the existing configurations.

The direct synthesis starts from the investigation of the mutual relations between the bond graph prototypes as shown in Fig. 5. And subsequently, it is used appropriately that impedances and admittances are just reciprocal in their functional roles, or equivalently, in their input-output behaviors. From the close examination of the bond graph prototypes, it is concluded to say that first, the $SY_{fb}(s)$ in Eq. (7) indicating the feedback expansion of a given admittance is nothing but the inverse of $SZ_{ff}(s)$ in Eq. (6) which represents the feedforward expansion of a given impedance. Second, the $SZ_{fb}(s)$ in Eq. (8) indicating the feedback expansion of a given impedance is nothing but the inverse of $SY_{ff}(s)$ in Eq. (5) which represents the feedforward expansion of a given admittance.

These results may be thought of as a matter of course and can be treated as trivial ones. However, they must be regarded as one of the most important principles in manipulating dynamic systems in the hardware level for their analysis and synthesis. They suggest that the structural transformation inside one physical system is always practicable without the help of a well-developed mathematical formalism. Bearing in mind the reciprocal relations of impedances and admittances and the above results, suppose a given impedance $SZ(s)$ is synthesized into feed-forward and feedback components like the forms in Fig. 5(d). By simply reversing the numerator and denominator of $SZ(s)$ we are now ready to apply the feedforward expansion technique to the corresponding admittance $SY(s)$. Note that the procedure for feedforward expansion is far more easier than that for feedback expansion as mentioned in the earlier section. Subsequently, the $SY(s)$ can be divided without any trouble by applying the PFE or the common denominator expansion (CDE) into two terms $Y(s)$ and $1/Z(s)$ which must be the forms of admittances and be attached together with $SY(s)$ through the 0-junction as shown in Fig. 5(a). In fact, the extraction into two admittances means that the feedback expansion of $SZ(s)$ itself makes the direct synthesis completed, except for determining which component of the two must be selected as feedforward one and then reversed, for example, $Z(s)$ in Fig. 5(d). This is why the second reverse of $SY(s)$ to retrieve the original impedance $SZ(s)$ gives the same effect as the structural transformation from Fig. 5(a) to Fig. 5(d) directly.

The conventional assignment of causality in bond graphs is often classified with the integral and derivative ones. In the modeling viewpoint, the method of assigning causality to an element to be considered is usually based on allowing the element to be independent element in order to immediately attain the state space model in the time domain. However, since the frequency-dependent impedance and admittance are deeply associated with the bond graph analysis and synthesis of dynamic systems, it is convenient to force the role of causality to indicate what signals are entered into and flowed out of the element. Thus, in case of analyzing and synthesizing a dynamic system with the bond graph prototypes, changes in the role of input and output are nothing but changes in the causality assignment, especially, in linear systems. Therefore either of the two components can even be reversed so that the component is able to determine the common variable (the effort variable in this case) on the feedback junction (the 0-junction), resulting into the bond graph with integral causality. If another component is reversed then the resulting bond graph is with derivative causality. Note that whatever kind of causality may be generated, both the bond graphs represent the same physical structure and dynamic behavior seen at the driving-port, which implies that there are only expressional gap between their governing equations such that the bond graph assigned with integral causality produces a set of differential equations and the bond graph with derivative causality yields a set of integral equations (Karnopp, 1983). On the other hand, the structural transformation between Fig. 5(c) and Fig. 5(b) is also used to synthesize a given admittance into feedforward and feedback components. In this case, the procedure similar to that stated just above is still serviceable.

Now, Let us evaluate the powerful strength of the direct synthesis relative to the analytical synthesis in applying to Eq. (12) of which we will mark the first term $Z_1(s)$ and the second $Z_2(s)$ as in Fig. 9(a). Reversing the $Z_1(s)$ and putting the CDE on the result yield

$$\frac{1}{Z_2(s)} = Y_2(s) = \frac{4s+5}{10} + \frac{1}{10s} \tag{19}$$

Assume that the $Y_2(s)$ in Eq. (19), the inverse of $Z_2(s)$, is expanded into $Z_3^{-1}(s) = (4s+5)/10$ and $Y_3(s) = 1/(10s)$. Note that Eq. (19) has the same structure as in Fig. 5(a). And $Z_3(s)$ is reversible and then becomes the feedforward component of $Z_2(s)$. From the impedance and admittance forms for the basic elements of bond graphs, $Y_3(s)$ can be recognized as inertia $I(s)$ with integral causality and $Z_3(s)$ as a sort of a dynamic system showing a damped-capacitance behavior. Again, since $Z_3(s) = 10/(4s+5)$ is too vague to be realized by the basic bond graph

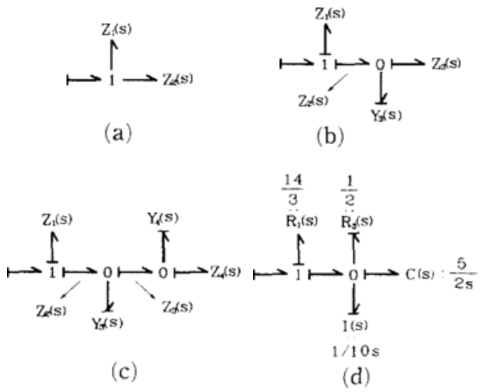

Fig. 9 History for the direct synthesis of Eq. (11)

elements, the procedure used to get Eq. (19) is applied to it, and the result is

$$\frac{1}{Z_3(s)}=\frac{2s}{5}+\frac{1}{2} \tag{20}$$

Let $Z_4^{-1}(s)$ be $2s/5$ and $Y_4(s)$ be $1/2$ in Eq. (20). This also means the $Z_4(s)=5/(2s)$ is the feedforward term of the $Z_3(s)$ as shown in Fig. 9 (c), where the $Z_4(s)$ plays the same role as capacitance $C(s)$ with integral causality and the $Y_4(s)$ as resistance in the admittance type. The final synthesis in which more general notations of bond graphs are inserted is in Fig. 9(d).

For the sake of contrast, consider that the roles of input and output are changed each other. If the $Y_2(s)$ in Eq. (19) has two terms such that $Z_3^{-1}(s)=1/(10s)$ and $Y_3(s)=(4s+5)/10$, then it can be easily found that the $Z_3(s)=10s$ becomes the feedforward element in the form of impedance and the admittance $Y_3(s)$ is again reticulated with synthetic division into another two feedforward terms $Y_4(s)=2s/5$ and $Y_5(s)=1/2$ as shown in Fig. 10(a). And the completed synthesis is in Fig. 10(b). Of course, both the structures in Fig. 9(d) and Fig. 10(b) describe the same physical system, for example, as in Fig. 8(a). It is no wonder that there are seemingly differences in the causality assignment and the forms of impedance and admittance. This is merely the results from the reflection of changing signals of input and output.

The direct synthesis is a very concise method in its usage and can be a rather scientific method in

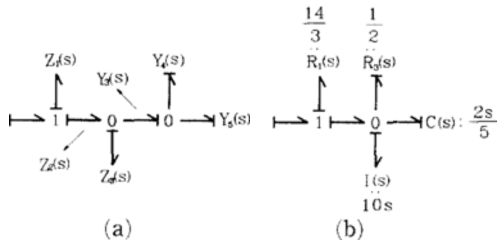

Fig. 10 Alternative history for the direct synthesis of Eq. (11)

managing bond graph prototypes systematically to obtain its reliable result. Therefore, if the direct synthesis developed in this paper is used to represent a given specification as the bond graph structure, it is obvious that the method removes some troublesome of selecting the coefficients in feedforward and feedback components and how to determine the synthesizing path, which are inherent attributions of the analytical synthesis, as stated in the earlier section.

5. Conclusions

Two dynamic system synthesis methods using bond graph prototypes have been proposed. These methods demonstrate some outstanding advantages compared to electrical network synthesis and any existing ones. The analytical synthesis shows its abilities in reticulating a given specification into feedforward and feedback components (we called this the 'feedback expansion' in this paper) relative to electrical network synthesis. And relative to an existing method like in the paper (Redfield et al., 1993), this method also exhibits its strengths in determining what types of the bond graph junctions and causalities must be selected to satisfy the dynamic behavior of a given specification physically. The second synthesis method developed in this paper was highlighted as the direct synthesis. This method not only rescues the tedious and annoying problems of choosing the coefficients in the reticulated components, but also gives some engineering intuitions into the physical system which is wholly described by the given specification.

All these benefits are due to the bond graph prototypes which might have a great effect in analyzing dynamic systems in the frequency domain, for example, in extracting transfer functions directly from them. The bond graph standards that Dr. Paynter originates in 1961 and that has been applied to various engineering systems by many researches are said to be modeling-oriented tools, while we would expect the bond graph prototypes to be analysis and synthesis oriented tools, at least after some years. To become so, further work of the bond graph prototypes is necessary to make them more refined and skilled as much as the bond graph standards have done, and to find their wide applications.

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